X100/301

NATIONAL QUALIFICATIONS 2003

WEDNESDAY, 21 MAY
9.00 AM – 10.10 AM

MATHEMATICS HIGHER

Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

LIB X100/301 6/3/26570

SCOTTISH QUALIFICATIONS AUTHORITY

ALL questions should be attempted.

Marks

1. Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$. **3**

2. (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. **2**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

3. Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$.
Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. **2**

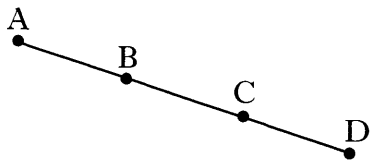
4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.

(*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2**

(*b*) Find the limit of this recurrence relation as $n \to \infty$. **2**

5. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

6. A and B are the points (–1, –3, 2) and (2, –1, 1) respectively.
B and C are the points of trisection of AD, that is AB = BC = CD.
Find the coordinates of D. **3**



7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

8. Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. **4**

9. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

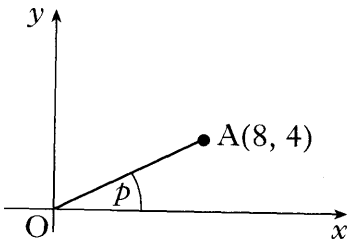
(*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

(*b*) Write down any restriction on the domain of h. **1**

[Turn over for Questions 10 to 12 on *Page four*

Marks

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

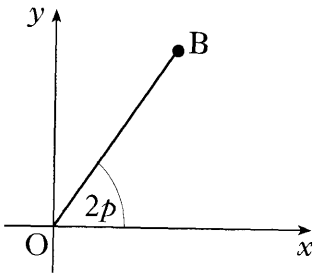


(*a*) Find the exact values of:

(i) $\sin(2p)$;

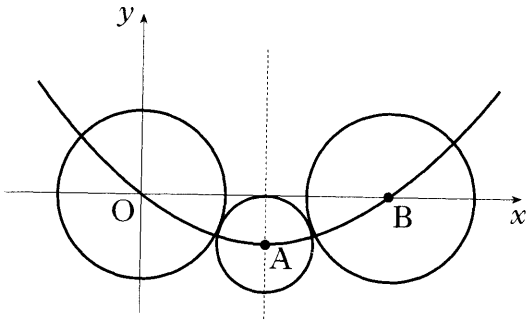
(ii) $\cos(2p)$. **5**

The line OB is inclined at an angle $2p$ radians to the x-axis.



(*b*) Write down the exact value of the gradient of OB. **1**

11.
- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y = px(x + q)$.
Find the values of p and q. **2**

12. Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. **4**

[END OF QUESTION PAPER]

X100/303

NATIONAL QUALIFICATIONS 2003

WEDNESDAY, 21 MAY
10.30 AM – 12.00 NOON

MATHEMATICS HIGHER

Units 1, 2 and 3
Paper 2

Read Carefully

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SCOTTISH QUALIFICATIONS AUTHORITY

ALL questions should be attempted.

Marks

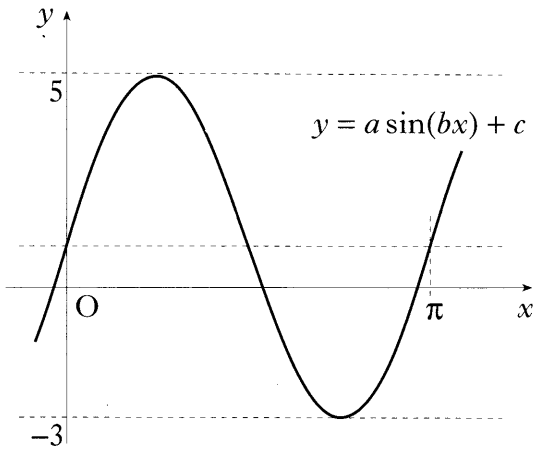
1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.

(*a*) Show that $(x - 2)$ is a factor of $f(x)$.

(*b*) Express $f(x)$ in its fully factorised form. **4**

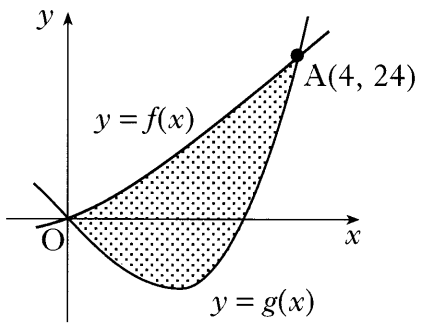
2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.

Determine the values of a, b and c. **3**



3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

Find the shaded area enclosed between the curves. **5**



4. (*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**

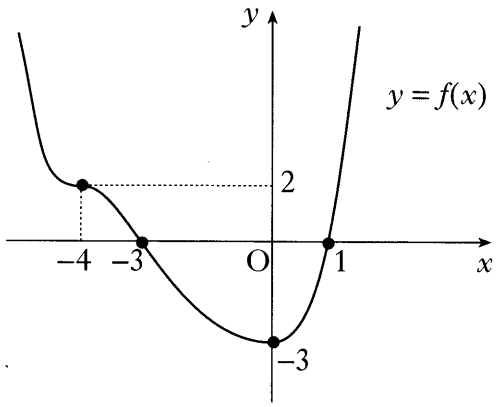
(*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**

[Turn over

Marks

5. The diagram shows the graph of a function f.

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

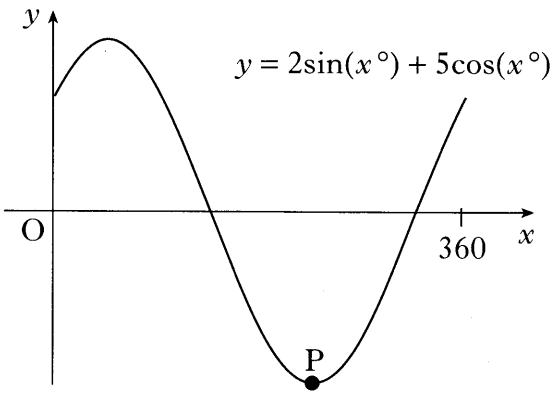


(a) Sketch the graph of $y = f(-x)$. **2**

(b) On the same diagram, sketch the graph of $y = 2f(-x)$. **2**

6. If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

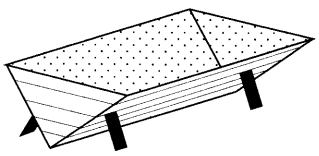
7. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.



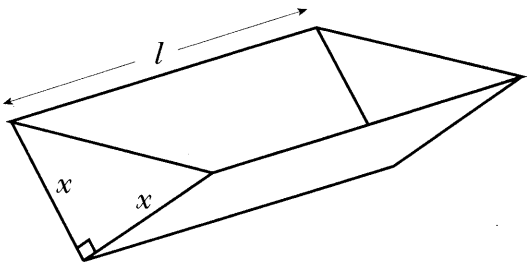
(a) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \leq a < 360$. **4**

(b) Find the coordinates of the minimum turning point P. **3**

8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.

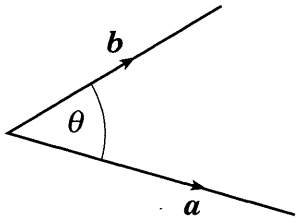


(a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3**

(b) Find the value of x which minimises this surface area. **5**

Marks

9. The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.
If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. **4**



10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places. **5**

11. (*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.
(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. **2**

(*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**

[END OF QUESTION PAPER]